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Modified Profitability Index and Internal Rate of Return

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Abstract

Both the internal rate of return (IRR) and the net present value(NPV) methods present well-known limitations. The drawbacks of the IRR include multiple rates, the assumption that cash flows are reinvested at the IRR, and the scale effect, whereas in the case of NPV the limitations relate to the choice of the measurement units as well as to the project's scale. In many cases, applying these two methods produces conflicting rankings for alternative investment projects. Two alternative models were developed to overcome these pitfalls: the modified internal rate of return method (MIRR), which overcomes the IRR's limitations, and the profitability index (PI), which resolves the limitations of NPV. The purpose of this paper is to show that there are no inconsistencies between the PI and MIRR, and that it is preferable to use a modified rate, the MPI, which is obtained by subtracting the cost of capital from the standard PI.

Keywords: modified internal rate of return, modified profitability index, project evaluation, capital investment decisions

1. Introduction

Two basic methods are used in investment analysis: the internal rate of return (IRR) and net present value (NPV). The former measures the return on the investment project while the latter measures the economic profit earned. In the IRR method, the return on the investment is measured in percentage terms by relating total profit – economic profit plus the opportunity cost of capital – to capital, whereas in the NPV method the economic profit is measured in financial value. Comparisons between the effectiveness of these methods in the evaluation of mutually exclusive investment projects have been the subject of widespread discussion in the research literature[4].

Despite their widespread use, both methods have well-known limitations: the IRR´s drawbacks include multiple rates, the assumption that cash flows are reinvested at the IRR, and the scale effect, whereas in the case of NPV the limitations relate to the choice of the measurement units and the project's scale. In many cases, applying these two methods produces conflicting rankings for alternative investment projects.

Two alternative models were developed in order to handle these shortcomings: the modified internal rate of return method (MIRR), which overcomes the IRR's limitations, and profitability index (PI), which deals with the NPV's drawbacks. When it comes to applying the profitability index, it is preferable to use a modified rate, the MPI, which is obtained by subtracting the cost of capital from the standard PI. This highlights a lack of consistency between the MPI and MIRR. With this modification, the MIRR consists of the sum of the MPI and the opportunity cost of capital.

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[4] See Cheng, Kite & Radke (1994), Copeland & Weston (1988), De Faro (1979).

2. Modified Internal Rate of Return - MIRR

The internal rate of return (IRR) is the total return, including the opportunity cost of capital earned by an investment project. By definition, we have:

$$P = \sum_{t=1}^{n} \frac{R_t}{(1+IRR)^t} \tag{1}$$

When evaluating conventional investment project with a steady cash flow (i.e., $R_t = R, \forall t$) one may rewrite formula (1) more concisely as:

$$P = R\left[\frac{(1+IRR)^n - 1}{IRR}\right]\frac{1}{(1+IRR)^n}$$

Then solve it:

$$(1+IRR)^n = \frac{R}{P}\left[\frac{(1+IRR)^n - 1}{IRR}\right] \tag{2}$$

The right side clearly shows that future net revenues are assumed to be reinvested at the IRR. Additionally, if the investment is non-conventional then the equation may yield more than one value for the IRR.

The MIRR method, whereby cash flows are reinvested at the firm's opportunity cost of capital, was created to overcome the limitations associated with the IRR. This procedure eliminates the potential issue of multiple IRR solutions, since in this case the cash flow exhibits a single inversion, and hence, due to Descartes's rule of signs, yields only one real positive root. In the case of a conventional investment project with a steady cash flow it is also possible to write:

$$P(1+MIRR)^n = R\left[\frac{(1+i)^n - 1}{i}\right] \tag{3}$$

The right side of the formula consists of the value of future income capitalized via rate *i,* the opportunity cost of capital – that is, the risk-free interest rate – whereas the left side corresponds to the future value of investments capitalized by the MIRR. The MIRR is thus the return on capital, and may be rewritten in the following way:

$$(1+MIRR)^n = \frac{R}{P}\left[\frac{(1+i)^n - 1}{i}\right] \tag{4}$$

Therefore, the MIRR depends on the opportunity cost of capital, the time horizon and the relationship between income and initial outlays, that is, the returns to scale. The higher the latter, the higher the MIRR will be. If the initial investment is doubled and the returns to scale are kept constant, then the MIRR will remain unchanged. This result matches those for the IRR and the MPI.

Consider the investment projects below:

A = {-1,000; 250; 250; 250; 250; 250} and B = {-2,000; 500; 500; 500; 500; 500}

Assuming an opportunity cost of capital of 5%, the MIRR is 6.675% for both investment projects, and lower than the IRR as the cash flows are reinvested at the firm's 5%opportunity cost of capital, which is lower than the IRR of 7.93%. This procedure successfully eliminates the IRR's main drawbacks. The MIRR for the entire period is 38.14%, which is precisely the total realized return.

3. Modified Profitabiliy Index – MPI

The NPV method[5] is the current value of future net revenues or the economic profit that a specific investment project earns, that is:

$$NPV = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t} - P \qquad (5)$$

Whereas:

NPV: net present value or economic profit
R_t: future net revenue or future profits
P: Initial investment outlay
i: Opportunity cost of capital
n: Project lifetime

To simplify the formula, we can assume that future net revenues will be constant and that we are dealing with a conventional investment project, that is, one in which there are no multiple sign changes in the cash flow stream.

Consequently, formula (5) can be rewritten concisely as:

$$NPV = R\left[\frac{(1+i)^n - 1}{i}\right]\frac{1}{(1+i)^n} - P \qquad (6)$$

One of the challenges in comparing IRR and NPV is that, while the IRR is a pure number – that is, the interest rate that equates the current value of input cash flows with the current value of output cash flows – the NPV is calculated in monetary terms and is therefore an absolute and not a relative measure of an investment project's worth. Usually the relative measure of profitability is calculated in relation to the initial capital outlay. The profitability index (PI) tries to solve this NPV limitation by comparing future net revenues discounted to present value. This enables comparisons between the project's economic profit and the initial investment outlay.

Analytically, the profitability index can be represented as:

$$PI = \frac{1}{P}\sum_{t=1}^{n} \frac{R_t}{(1+i)^t} \qquad (7)$$

which in the case of future and constant net revenues becomes:

$$PI = \frac{R}{P}\left[\frac{(1+i)^n - 1}{i}\right]\frac{1}{(1+i)^n} \qquad (8)$$

By replacing (8) with (6) it is possible to determine the relationship between the NPV and the PI:

$$PI = 1 + \frac{NPV}{P} \qquad (9)$$

Analytically, the relationship between the IRR, NPV and PI is:

[5] Economic profit should not be mistaken for accounting profit. It is the value earned above the opportunity cost of capital.

Where NPV = 0 then PI = 1 and IRR = i
Where NPV > 0 them PI > 1 and IRR > i
Where NPV < 0 then PI < 1 and IRR < i

The profitability index, which is calculated by dividing future net revenues or future economic profit brought to present value by the cost of capital, can be regarded as an undiscounted profit margin that includes the cost of used resources. By discounting the cost of capital, formula (9) yields a modified PI measure, MPI (10):

$$MPI = \frac{P + NPV - P}{P} = \frac{NPV}{P} \tag{10}$$

Therefore, the modified profitability index (MPI) corresponds exactly to the profit margin of the investment project since it relates the NPV– or actual economic profit– with the initial investment. We can better understand this adjustment by looking at an example.

Consider the previous example of two distinct investment projects:

A = {-1,000; 250; 250; 250; 250; 250} and B = {-2,000; 500; 500; 500; 500; 500}

Project A has an IRR equal to 7.93% and a NPV of R$ 82.36, while project B has an IRR equal to 7.93% and a NPV of R$ 164.73 for an opportunity cost of capital of 5% per period.

Note that the NPV will increase twofold as a result of the scale of investment being doubled, although the profitability of both projects will remain unchanged. This is a result of the NPV being an absolute and not a relative measure. In addition, it should be noted that when an NPV analysis compares different investment projects, it assumes that there are no restrictions concerning the initial investment. If this is the case, then project B would be chosen as it provides the same return as project A while generating higher profit.

The profitability index fixes this distortion presented by the NPV, showing that the two projects have the same PI:

Project A: *PI = 1 + (82.36/1,000) = 1.08236*

Project B: *PI = 1 + (164.73/2,000) = 1.08236*

Therefore, the PI solves the scale effect by using a dimensionless measure, as is the case for the IRR. In either project, there is an 8.236% return over the opportunity cost of capital after deducting the cost of capital:

Project A: MPI = (82.36/1,000) = 8.236%

Project B: MPI = (164.73/2,000) = 8.236%

Thus, the total return π of both investment projects is given by:

$$\pi = [(1+0.08236)(1+0.05)^5 - 1] = 38.14\%$$

Compared to a cumulative IRR of 46.46%. The IRR is higher as a result of future net revenues being reinvested at the IRR, which is higher than the opportunity cost of capital.

The total return is:

$$\pi = [(1 + MPI)(1 + i)^n - 1] \tag{11}$$

From formula (10) the MPI equals:

$$MPI = \frac{R}{P}\left[\frac{(1+i)^n - 1}{i}\right]\frac{1}{(1+i)^n} - 1 \tag{12}$$

The MPI depends on the opportunity cost of capital, the time horizon and the relationship between revenues and initial investment, that is, the returns on scale. The higher the latter, the higher the rate of return will be. If constant returns on scale are achieved as a result of the initial investment being doubled, then the profitability index will remain unchanged while the NPV will increase twofold. In this case, the PI solves the drawback presented by the NPV.

4. Relationship between MIRR and MPI

The relationship between the concepts of MIRR and MPI will now be examined. Replacing formula (8) with (4):

$$(1+MIRR)^n = (1+MPI)(1+i)^n \tag{13}$$

In this equation, the MIRR and the opportunity cost of capital are measured per period, while the MPI is cumulative for the investment project's entire lifetime. Should one prefer to measure all three variables per period, the following formula would be:

$$(1+MIRR) = (1+MPI)(1+i) \tag{14}$$

Thus, for small values it is possible to apply the following approximation:

$$MIRR \cong MPI + i$$

We conclude that the MIRR consists of the MPI and opportunity cost of capital, and that there are no inconsistencies between these two indices. For a given opportunity cost of capital, the greater the MPI the greater the MIRR will be, and vice-versa. Therefore, there is an increasing monotonic relationship between these two variables. As a result, the use of either of the two indicators leads to the same result and rankings for different investment projects.

5. Conclusions

The comparison between the effectiveness of the IRR and NPV investment analysis methods has been the subject of extensive discussions. Despite their widespread use, both methods have well-known limitations: the IRR´s drawbacks include multiple rates, the assumption that cash flows are reinvested at the IRR, and the scale effect, whereas in the case of NPV limitations relate to the choice of the measurement units and the project's scale. In many cases, applying these two methods produces conflicting rankings for alternative investment projects. The modified internal rate of return (MIRR) and the profitability index (PI) are two alternative models developed to overcome these drawbacks. In the MIRR method, the cash flows are reinvested at the firm's opportunity cost of capital, whereas in the PI method the NPV refers to the initial capital outlay. When it comes to applying the profitability index, it is preferable to use a modified rate, the MPI, which is calculated by subtracting the cost of capital from the standard PI.

As a result, there is a linear relationship between the MIRR and MPI, and no inconsistencies exist between them. With this modification, the MIRR consists of the MPI plus the opportunity cost of capital. The first advantage here concerns the simplicity of the resulting equation: $MIRR = MPI + i$. additionally, these two methods produce the same rankings with regards to ranking alternative investment projects.

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